Learning Objective

- Understand how to find a basis for Row A
- Use the Rank Theorem to determine the dimensions of a subspace
- Understand how the Rank Theorem relates to solving a system of linear equations

The Row Space

Definition: If A is an $m \times n$ matrix, the ___________________ of A, denoted by Row A, is the set of all ____.

Remarks:

- Each row has ______ entries, so Row A is a subspace of _________.
- The row space of A is the same as the column space of _________.

Example: Let $A = \begin{bmatrix} -1 & 2 & 3 & 6 \\ 2 & -5 & -6 & -12 \\ 1 & -3 & -3 & -6 \end{bmatrix}$. Find a spanning set for the row space of A.

Theorem 4.13: If two matrices A and B are row equivalent, then their __________ ________________ are the same.

If B is in echelon form, then the nonzero rows of B form a ________________ for both ____.

Example: Find bases for the row space, the column space, and the null space of $A = \begin{bmatrix} -1 & 2 & 3 & 6 \\ 2 & -5 & -6 & -12 \\ 1 & -3 & -3 & -6 \end{bmatrix}$.

Note that $A \sim B = \begin{bmatrix} -1 & 2 & 3 & 6 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

The Rank Theorem

Definition: If A is an $m \times n$ matrix, the __________ of A is the ________________________ of the column space of A.

Remark: Since Row A is equal to Col A^T, the dimension of the row space of A is the rank of A^T.

Theorem 4.14 (The Rank Theorem): The dimensions of the column space and the row space of an $m \times n$ matrix A are ______________. This common dimension, the rank of A, equals the number of _______________________________ in A and satisfies the equation

$$\_\_\_\_\_\_\_\_\_\_ + \_\_\_\_\_\_\_\_\_\_\_\_ = \_\_\_\_$$

Remark: The dimension of the null space is sometimes called the **nullity** of A and Theorem 4.14 is sometimes referred to as the Rank-Nullity Theorem.

Example: Let A be a 5×8 matrix.

(a) Find the smallest possible value of $\dim \operatorname{Nul} A$.

(b) If rank $A = 5$, find $\dim \operatorname{Nul} A$, $\dim \operatorname{Row} A$, and rank A^T.