## Learning Objectives

- Understand how row operations change the determinant of an $n \times n$ matrix
- Understand how the determinant of an $n \times n$ matrix is related to its invertibility

## Properties of Determinants

**Theorem 3.3: Determinants and Row Operations**

Let $A$ be a square matrix.

(a) If a multiple of one row of $A$ is added to another row to produce a matrix $B$, then $\det B =$ ______________.

(b) If two rows of $A$ are interchanged to produce $B$, then $\det B =$ ______________.

(c) If one row of $A$ is multiplied by $k$ to produce $B$, then $\det B =$ ______________.

**Example:** Use row reduction to compute $\det \begin{bmatrix} 2 & 4 & 6 \\ 5 & 6 & 7 \\ 4 & 8 & 4 \end{bmatrix}$.

**Theorem 3.4:** A square matrix $A$ is invertible if and only if __________________________.

**Example:** Compute $\det A$ and determine if the matrix $A = \begin{bmatrix} 1 & 3 & -2 & -2 \\ -1 & 0 & 3 & 2 \\ 0 & 1 & 1 & 0 \\ 2 & 6 & 0 & -4 \end{bmatrix}$ is invertible.

**Theorem 3.5:** If $A$ is an $n \times n$ matrix, then $\det A^T =$ ______________.

*Remark:* Theorem 3.5 says Theorem 3.3 holds if the word <u>*row*</u> is replaced with __________________.

**Theorem 3.6:** If $A$ and $B$ are $n \times n$ matrices, then $\det AB =$ ________________________.

**Example:** Let $A = \begin{bmatrix} 3 & 0 \\ 6 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 \\ 5 & 4 \end{bmatrix}$.

(a) Verify that $\det AB = \det A \det B$.

(b) Verify that $\det(A + B) \neq \det A + \det B$.