Definition: Assume that A is an $m \times n$ matrix that can be row reduced to echelon form *without row interchanges.* Then A can be written in the form

where L is an $m \times m$ lower triangular matrix with 1's on the diagonal and U is an $m \times n$ (upper triangular) echelon form of A.

Such a factorization is called an ____ of A.

Why are LU factorizations useful?

LU factorizations improve the computational efficiency of solving matrix equations.

If $A = LU$, then the matrix equation $A\mathbf{x} = \mathbf{b}$ can be rewritten as

$$L(U\mathbf{x}) = \mathbf{b}$$

To solve $A\mathbf{x} = \mathbf{b}$, we can solve the pair of equations:

2. Let $A = \begin{bmatrix} 3 & 0 & 1 \\ 3 & 1 & 1 \\ 3 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = LU.$

Use this LU factorization to solve $A\mathbf{x} = \mathbf{b}$ where $\mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$.

Step 1: Solve $L\mathbf{y} = \mathbf{b}$.

Step 2: Solve $U\mathbf{x} = \mathbf{y}$.

Conclusion: