Quiz 4 solutions

1. 1) 2

   2) 3

   3) 2

   4) is linearly dep't and may or may not span

   5) may or may not be linearly indep't.

2. $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ (eigenvalue -1)

   $\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \end{bmatrix}$ (eigenvalue 2)

3. $\lambda=2$ eigenspace $=$ Null$(\cancel{\lambda}C-2I)$

   $= \text{Null}\left(\begin{bmatrix} -1 & 1 & 2 \\ 1 & -1 & -2 \\ 1 & -1 & -2 \end{bmatrix}\right)$

$R2+R1$
$R3+R1$ $\hookrightarrow$ $\begin{bmatrix} -1 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$-R1$ $\hookrightarrow$ $\begin{bmatrix} 1 & -1 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$$x_1 = x_2 + 2x_3$$
x_2, x_3 free

$$\begin{bmatrix} x_2 + 2x_3 \\ x_2 \\ x_3 \end{bmatrix} = x_2 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

$$\leadsto \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \right\}.$$

4. $[\underline{x}]_\beta = \begin{bmatrix} 2 \\ -1 \end{bmatrix} \rightsquigarrow \underline{x} = 2\underline{b_1} + (-1)\underline{b_2}$

$$= 2\begin{bmatrix} 1 \\ 0 \\ -1 \\ -2 \end{bmatrix} + -1 \cdot \begin{bmatrix} 2 \\ 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ -2 \\ -3 \end{bmatrix}.$$

$$\begin{bmatrix} -1 \\ -3 \\ -5 \\ -7 \end{bmatrix} = \underline{a_1}\,\underline{b_1} + a_2\,\underline{b_2} \rightsquigarrow [\underline{x}]_\beta = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}.$$

linear system

$$\left[\begin{array}{cc|c} 1 & 2 & -1 \\ 0 & 1 & -3 \\ -1 & 0 & -5 \\ -2 & -1 & -7 \end{array}\right] \rightsquigarrow [\underline{x}]_\beta = \begin{bmatrix} 5 \\ -3 \end{bmatrix}.$$

5. $D\begin{bmatrix}0\\1\\3\end{bmatrix} = \underline{b}$, $D\underline{x} = \underline{b} \Leftrightarrow D\left(\begin{bmatrix}0\\1\\3\end{bmatrix} - \underline{x}\right) = \underline{0}$.

$$\begin{bmatrix}0\\1\\3\end{bmatrix} - \begin{bmatrix}1\\2\\4\end{bmatrix} = \begin{bmatrix}-1\\-1\\-1\end{bmatrix} \in \text{Span}\left\{\begin{bmatrix}-1\\0\\1\end{bmatrix}, \begin{bmatrix}2\\1\\0\end{bmatrix}\right\}$$

(since $\begin{bmatrix}-1\\-1\\-1\end{bmatrix} = a\begin{bmatrix}-1\\0\\1\end{bmatrix} + b\begin{bmatrix}2\\1\\0\end{bmatrix}$ has the solution $a=-1$, $b=-1$)

~~So~~ $\underline{x} = \begin{bmatrix}1\\2\\4\end{bmatrix}$.