# Linear Algebra (Math 314) – Project 1
Due Friday, October 2

Instructions: The following project is based on sections 1.6, 1.10, and 2.7 of our text. It covers a few elementary applications of linear systems, most of which were not covered in the lectures. You should carefully read through the sections and pay particular attention to the examples presented in the text. After reading the sections, complete the three problems included below. *You must show all relevant work on all problems in order to get full credit.* You may use a computer or calculator to compute the RREF of matrices. You may work in groups of 2–3 students and you need only submit one project per group, with all the names of the group members at the top.

1. **Balancing chemical equations** - Section 1.6
   Limestone, $CaCO_3$, neutralizes the acid H_3O in acid rain by the following unbalanced equation:
   $$H_3O + CaCO_3 \longrightarrow H_2O + Ca + CO_2.$$
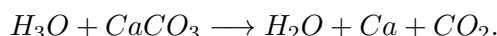
   (a) Set up a vector equation that describes the number of atoms of each type present in this reaction. Find the general solution of this vector equation.

   (b) Find the balanced equation if the reaction produced exactly 1 molecule of CO_2. What is the balanced equation if the reaction produces 6 molecules of CO_2?

   (c) Is it possible to have a balanced equation involving only 1 molecule of H_3O?

2. **Difference Equations** - Section 1.10

   (a) In a city, 7% of the city's population moves to the surrounding suburbs each year and 5% of the suburban population moves into the city. In 2010, there were 800,000 people living in the city and 600,000 living in the suburbs. Set up a difference equation that describes this situation, where x_0 is the population in 2010.

   (b) Use the difference equation to find the populations in the city and the suburbs in 2011, 2012.

   (c) What are the likely populations of the city and suburbs in 2020?

   (d) The University of Nebraska system consists of three campuses: Lincoln, Omaha, and Kearney. Data was collected regarding the transfer rates of students among campuses. The data is as follows (rounded to the nearest percent): 5% of Lincoln students transfer to Omaha while 2% transfer to Kearney; 3% of Omaha students transfer to Lincoln while 0% transfer to Kearney; and 1% of Kearney students transfer to Lincoln while 0% transfer to Omaha. The 2020 enrollments for the campuses are 26,000 (Lincoln), 15,000 (Omaha), and 6,000 (Kearney). Set up a difference equation which describes this situation, where x_0 is the enrollment in 2020, and use this to estimate the enrollment at the three campuses in 2025 and 2026. (Assume that incoming freshmen, graduating students, etc. cancel themselves out).

3. **Computer Graphics** - Section 2.7

    (a) Consider the triangle S in $\mathbb{R}^2$ with vertices $(0, 1)$, $(-2, -1)$, and $(2, -1)$. Determine homogeneous coordinates for the vertices of S. (These will each have 3 entries.)

    (b) Find the 3×3 matrix that produces the following composite 2D transformation: translation by $(2, 1)$, followed by rotation $90°$ about the origin, and then scaling the x-coordinate by 2 and the y-coordinate by 3.

    (c) Use this matrix to determine the image of S in $\mathbb{R}^2$ under the composite transformation in (b). Provide a sketch of this new triangle in the plane.