Habits of Mind

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"My methods are really methods of working and thinking; this is why they have crept in everywhere anonymously."

- *Emmy Noether*

1 The Mathematical Thinker

In Mathematics education we are often presented with algorithmic methods for solving certain problems. The challenge is then to apply these algorithms in a technically correct way to arrive at a solution. This experience would seem to imply there is little room for artistry in Math. However, these methods have not always existed. They are the direct product of centuries of careful thought and trenchant insights. So what does the process of arriving at these ideas look like? In other words: what do we do when we don't know what to do?

In this activity, we will consider several problems designed to encourage creative problem solving. Though mathematical facts and information from past experiences may prove useful, each problem will ultimately require careful thought (and often: discussion).

2 Cents and Sensibility

What is the fewest number of coins that it will take to make 43 cents if you have available pennies, nickels, dimes and quarters? After you have solved this problem, provide an explanation that proves that your answer is correct. How does the answer (and the justification) change if you only have pennies, dimes and quarters available?

3 Four Fours

Using the symbol "4" exactly four times, write expressions for the natural numbers between one and twenty, using only the Mathematical symbols listed below:

1. Arithmetic operations
2. Parentheses
3. Decimal points or percentages
4. Exponentiation
5. Square roots
6. Factorials
7. Concatenation

Example 1 $1 = (\frac{4}{4})^{44}$

Note: there are many different ways to do this for each number. Try to find a few, or compare your results with classmates.

4 How to Cut Pizza the Wrong Way

Consider a large circle (pizza) and pick n points on the circle (the crust). Here, n might be 2, 3, 4, and so forth. Connect each pair of points with a straight line. Notice that if $n = 2$ the circle is cut into 2 regions (slices). If $n = 3$ the circle is cut into 4 regions and if $n = 4$ the circle is cut into 8 regions. How many regions do you get if $n = 6$? What about $n = 8$?

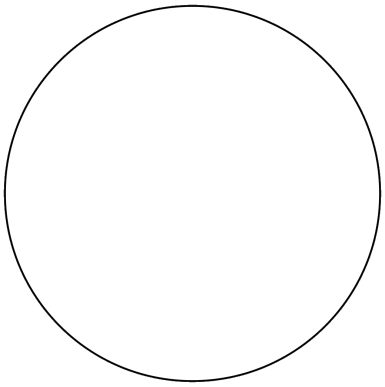

Figure 1: "Pizza"

5 Crazy Eights

How many ways are there to use copies of the symbol "8" strung together with plus (+) signs to equal 1000? Once you think you have an answer, can you provide a mathematical argument that you are correct?

Example 2 $888 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 = 1000$