

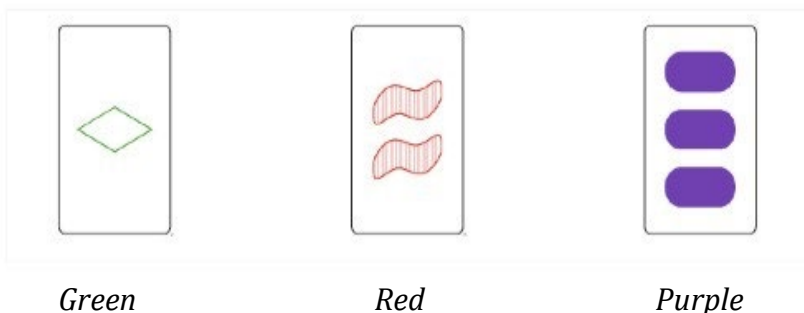
## The Mathematics of SET

### SET cards:

Each card has either one, two, or three shapes. All the shapes on a given card are the same, and share the same color and shading.

The possible shapes are diamond, squiggle, oval; the possible colors are red, green, and purple; the possible shadings are plain, striped, and solid.

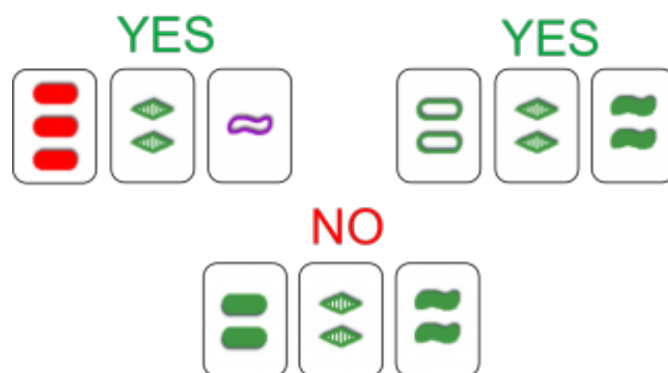
So each card has four characteristics: **NUMBER, SHAPE, COLOR, SHADE**. For example, here are the cards "one plain green diamond," "two striped red squiggles," and "three solid purple ovals."



There is exactly one of each possible card in the SET deck.

1. How many cards are there in a SET deck?

2. The three cards above form a SET. Why?



**SET rules:**

A SET is a collection of three cards such that for each characteristic, either all of the cards in the trio share the same value for that characteristic, or else no two have the same value. For example, if the characteristic is SHAPE, then the three cards should all have the same shape, or they should all have different shapes. So we need to check each characteristic to determine if a trio is a SET.

The game is played by dealing an array of 12 cards in the middle of the group. The first one to spot a SET in the array shouts "SET" and collects the three cards; these cards are replaced and play continues. The person with the most SETs at the end wins. If there is no set amongst the 12 cards, three more cards are dealt.

**3. How many different SETs are there?**

**4. Show that for every two cards, there is a unique third card that will make a SET.**

**5. If you draw three cards at random, what is the probability that you will get a set?**

**6. Play through an entire SET deck, except keep the last card face down. Can you guess what is on the card? How do you know?**

### Coordinatization of the SET deck:

Let's assign each card in a SET deck with a string of 4 numbers, each number representing one of the characteristics.

Coordinate	Number	Shading	Color	Shape
1	1	plain	red	diamond
2	2	striped	green	oval
0	3	solid	purple	squiggle

So, for example, the card "2, striped, green, diamonds" becomes the point (2,2,2,1) using this coordinate system.

To add or subtract in our system, we can go coordinate by coordinate. For example,

$$(2,2,1,0) + (2,1,2,2) = (2 + 2, 2 + 1, 1 + 2, 0 + 2) = (4,3,3,2).$$

It will be more convenient if, when we add and subtract cards, we end up with something in our coordinate system. *So we will use addition and subtraction modulo 3.* This just means that after we add, we divide the answer by 3 and use the remainder as our solution. So we can write

$$(4,3,3,2) = (1,0,0,2).$$

Here is an addition table mod 3:

+	-2	-1	0	1	2
-2	2	0	1	2	0
-1	0	1	2	0	1
0	1	2	0	1	2
1	2	0	1	2	0
2	0	1	2	0	1

**7. Add the coordinates of any three cards in a SET. What do you get? Is the answer always the same?**

You can play a daily SET puzzle online at <https://www.setgame.com/set/puzzle>.